

# Interacting Holographic Viscous Dark Energy Model

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Received: 4 August 2009 / Accepted: 16 October 2009 / Published online: 27 October 2009  
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**Abstract** In this manuscript, we present a generalization of the interacting holographic dark energy model using the viscous generalized Chaplygin gas. We also study the model by considering a dynamical Newton's constant  $G$ . Then we reconstruct the potential and the dynamics of the scalar field which describe the viscous Chaplygin cosmology.

**Keywords** Dark energy · Dark matter · Varying gravitational constant

## 1 Introduction

Astrophysical observations suggest that the observable universe is undergoing in a transition from the earlier deceleration phase (matter dominated era) to the acceleration phase (dark energy dominated era) [1–5]. The empirical results based on the data are  $\Omega_m \approx 0.3$  while  $\Omega_\Lambda \approx 0.7$ . The simplest explanation to dark energy phenomenon is the cosmological constant  $\Lambda$  which satisfies the equation of state (EoS)  $p = -\rho$  or  $\omega = -1$  ( $p = \omega\rho$ ) where  $p$  is pressure and  $\rho$  is the energy density. If the universe is dominated by  $\Lambda$  than it will maintain constant energy density and will dominate matter rapidly at some instant in the evolution of the universe. Although the cosmological constant offers a solution to the dark energy problem but with several drawbacks like fine tuning and cosmic coincidence problem. The former one requires fine tuning of the energy density of dark energy to match the theoretical and observational values. The later problem arises since it is unlikely that the current transition period coincides with the current time. In other words, why the energy densities of matter and dark energy are so much comparable at the current epoch.

One of the promising resolutions to the dark energy problem is the model based on the dark energy-dark matter (DE-DM) interaction and it presents a possible resolution to the

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above problems [6–18]. The interaction is assumed to be negligible at high redshifts while it is large at lower redshifts, thus it is motivating to make observations to detect the said interaction [19]. A possible way to alleviate the coincidence problem is to suppose that there is an interaction between matter and dark energy. The cosmic coincidence can then be alleviated by appropriate choice of the form of the interaction between matter and dark energy leading to a nearly constant ratio  $r_m \equiv \rho_m / \rho_{de}$  during the present epoch or giving rise to attractor of the cosmic evolution at late time. In the current model, it can be rephrased as why the interaction rate is of the order of the Hubble rate at present epoch. Recently some tracker solutions are obtained for the DE-DM model which show that once the attractor for the system is reached, the ratio between the corresponding energy densities remains constant afterwards, thereby solving the coincidence problem [20–22]. It is also recently studied the dark energy decay into matter at the Hubble rate which is a good fit with the observational data supporting an accelerating universe [23].

In this paper, we offer a connection between the holographic viscous dark energy and the interacting dark energy. The former model is also an alternative proposal to the problem of cosmic accelerated expansion. In the second last section, we extend our study by considering a dynamical Newton’s constant  $G$ . Finally we conclude our paper.

## 2 The Interacting Model

We start by assuming the background to be spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetime, specified by the line element:

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \tag{1}$$

where  $a(t)$  is the scale factor and  $k$  is the curvature parameter. For the sake of generality, we shall assume  $k$  to be different from zero. The corresponding Einstein field equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} [\rho_{de} + \rho_m]. \tag{2}$$

Here  $M_p^2 = (8\pi G)^{-1}$  is the reduced Planck mass. We assume matter and dark energy interacting each other, then the energy conservation equations read

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q, \tag{3}$$

$$\dot{\rho}_m + 3H\rho_m = Q. \tag{4}$$

Here overdot represents differentiation with respect to cosmic time  $t$ . In explicit form, we have  $p_{de} = \omega_{de}\rho_{de}$  and  $p_m = 0$  (or  $\omega_m = 0$ ). Note that the subscripts  $de$  and  $m$  refer to dark energy and matter respectively. Due to energy transfer, local energy conservation will not hold but for the whole interacting system, thus interaction leads to a modification of the standard  $\Lambda$ CDM model. This interaction is naturally expected if the two species exist in dominant quantities. It is generally assumed that baryons don’t interact dark energy and dark matter can. Since both dark energy and dark matter are largely unknown, therefore the precise expression for the interaction would be largely hypothetical. Here  $Q(\alpha_m\rho_m, \alpha_{de}\rho_{de}) \simeq \alpha_m\rho_m + \alpha_{de}\rho_{de}$  is the interaction term which is a function of densities and two coupling parameters corresponding the interacting components [24]. It determines

the rate of change of energy in the unit comoving volume. The direction of transfer of energy depends on the sign of  $Q$  i.e. positive  $Q$  represents energy transfer from dark matter to dark energy and vice versa for negative  $Q$ . Since more parameters make the model to be less and less predictive, so we shall use  $\alpha_m = \alpha_{de} = b^2$  [25]. In [26], it is suggested that interaction term should be proportional to the number densities of the interacting medium to get a physically interesting interaction term. The interacting model also best fits the data of luminosity distance of supernovae of type Ia and with the WMAP observations of cosmic microwave background [27–30]. These observations constrain the interacting parameter  $b^2 < 10^{-2}$  at  $3\sigma$  level. Moreover,  $Q = 3Hb^2(\rho_m + \rho_{de})$  is the explicit form of interaction will be used here onwards. Here  $3H$  is attached for dimensional consistency.

Here we define the effective equations of state for dark energy and matter as [31]

$$\omega_{de}^{eff} = \omega_{de} + \frac{\Gamma}{3H}, \quad \omega_m^{eff} = -\frac{1}{r_m} \frac{\Gamma}{3H}. \tag{5}$$

Here  $\Gamma = Q/\rho_{de}$  is the decay rate. Making use of (5) in (3) and (4) yields

$$\dot{\rho}_{de} + 3H(1 + \omega_{de}^{eff})\rho_{de} = 0, \tag{6}$$

$$\dot{\rho}_m + 3H(1 + \omega_m^{eff})\rho_m = 0. \tag{7}$$

The dimensionless density parameters corresponding to matter, dark energy and curvature are

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3H^2 M_p^2}, \quad \Omega_{de} = \frac{\rho_{de}}{\rho_{cr}} = \frac{\rho_{de}}{3H^2 M_p^2}, \quad \Omega_k = \frac{k}{(aH)^2}. \tag{8}$$

Here  $\rho_{cr} \equiv 3M_p^2 H^2$  is the critical density. Observations indicate that universe is spatially flat but after the inclusion of higher order corrections, spatial curvature enters the luminosity distance of SN Ia supernova. In this connection, it is demonstrated that the reconstruction of the EoS of dark energy can lead to gross errors [32].

In an isotropic and homogeneous FRW universe, the dissipative effects arise due to the presence of bulk viscosity  $\xi$  in cosmic fluids. The theory of bulk viscosity was initially investigated by Eckart [33] and later on pursued by Landau and Lifshitz [34]. Dark energy with bulk viscosity has a peculiar property to cause accelerated expansion of phantom type in the late evolution of the universe [35–37]. It can also alleviate several cosmological puzzles like age problem [38], coincidence problem [39] and phantom crossing [40]. A viscous dark energy EoS is specified by

$$p_{eff} = p_{de} + \Pi. \tag{9}$$

Here  $\Pi = -\xi(\rho_{de})u^\mu_{;\mu}$  is the viscous pressure and  $u^\mu$  is the four-velocity vector. We require  $\xi > 0$  to get positive entropy production in conformity with second law of thermodynamics [41–43]. In FRW model, it takes the form  $\Pi = -3H\xi$  [34, 44], so that

$$p_{eff} = \frac{\chi}{\rho_{de}^\alpha} - 3H\xi(\rho_{de}). \tag{10}$$

The first term on the right hand side is called the generalized Chaplygin gas with  $0 < \alpha \leq 1$ . It reduces to the Chaplygin gas if  $\alpha = 1$  and converts to polytropic case if  $\alpha < 0$ . In general,  $\xi(\rho_{de}) = \nu\rho_{de}^s$ ,  $\nu \geq 0$ , where  $s$  and  $\nu$  are constant parameters. In particular, for the case,  $s = 1/2$  i.e.  $\xi(\rho_{de}) = \nu\rho_{de}^{1/2}$ , yields a power-law expansion for the scale factor [45, 46]. Moreover, if we demand to have the occurrence of a big rip in the future cosmic time then we

have the following constraint on the parameter  $v$ :  $\sqrt{3}v > \beta$ , where  $\beta \equiv 1 + \omega_{de}$ , leading the scale factor to blow up in a finite time [47]. We assume the parametric form  $\xi(\rho_{de}) = v\rho_{de}^{1/2}$ . Hence (10) becomes

$$p_{eff} = \frac{\chi}{\rho_{de}^\alpha} - 3vH\rho_{de}^{1/2}. \tag{11}$$

Use of (11) in the energy conservation equation,  $\dot{\rho}_{de} + 3H(\rho_{de} + p_{eff}) = 0$ , yields

$$\rho_{de} = \left[ \frac{Da^{-3(1+\alpha)(1-v\gamma)} - \chi}{1 - v\gamma} \right]^{\frac{1}{1+\alpha}}. \tag{12}$$

Here  $D$  is a constant of integration,  $\gamma = M_p^{-1}\sqrt{1 - r_m}$ , where  $r_m \equiv \rho_m/\rho_{de} = \Omega_m/\Omega_{de}$ . In the absence of interaction,  $r_m$  will be decreasing with time and increasing in the case of interaction. The effective EoS of dark energy is given by [48]

$$\omega_{de}^{eff} = -\frac{1}{3} - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c}. \tag{13}$$

It is now clear that the current accelerated expansion is not the first time in the expansion history of the universe rather it was earlier preceded by cosmic inflation. The latter was supposedly driven by a dynamically evolving scalar field commonly called inflaton [49]. It has motivated to develop scalar field models dealing with minimally coupled scalar field. Hence it can be anticipated that the current accelerated expansion is driven by a similar dynamical scalar field  $\phi$  with potential  $V(\phi)$ , related to the energy density and the pressure of viscous dark energy as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left[ \frac{Da^{-3(1+\alpha)(1-v\gamma)} - \chi}{1 - v\gamma} \right]^{\frac{1}{1+\alpha}}. \tag{14}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{\chi}{\left[ \frac{Da^{-3(1+\alpha)(1-v\gamma)} - \chi}{1 - v\gamma} \right]^{\frac{\alpha}{1+\alpha}}} - 3vH \sqrt{\left[ \frac{Da^{-3(1+\alpha)(1-v\gamma)} - \chi}{1 - v\gamma} \right]^{\frac{1}{1+\alpha}}}. \tag{15}$$

Until recently, the interaction of dark energy based on the holographic principle has been introduced to explain the coincidence problem and phantom crossing scenario (the transition quintessence to the phantom phase or the phantom non-phantom transition) [50–55]. The principle is based on the idea that all the information contained inside a spatial volume can also be obtained from the information present on its surface (see [56, 57] for comprehensive review). The principle has emerged from the quantum gravity of black holes. We shall follow the formulation of Cohen et al. [58] who proposed that in quantum field theory a short distance (UV) cut-off is related to a long distance (IR) cut-off due to the limit set by forming a black hole. In other words, if the quantum zero-point energy density  $\rho_{de}$  is relevant to a UV cut-off, the total energy of the whole system with size  $L$  should not exceed the mass of a black hole of the same size, thus we have  $\rho_{de}L^3 \leq M_p^2L$  [59–63]. After saturating this inequality with the largest IR cut-off, we obtain the energy density of the holographic dark energy is represented by

$$\rho_{de} = 3c^2M_p^2L^{-2}, \tag{16}$$

Here  $c$  is a small positive constant of order unity. From the information of supernovae SN Ia and cosmic microwave background radiation, it is deduced that the holographic parameter has the constraint  $c = 0.81^{+0.23}_{-0.16}$  [64]. Also  $L$  is the infrared cut-off which can be taken as

$$L = ar(t). \tag{17}$$

In the cosmological context, the largest IR cut-off can be taken as the size of the Hubble horizon,  $L = H^{-1}$ . Using the FRW metric, one can obtain [65, 66]

$$L = a(t) \frac{\text{sinn}[\sqrt{|k|}R_h/a(t)]}{\sqrt{|k|}}, \tag{18}$$

where  $R_h$  is the size of the future event horizon defined as

$$R_h = a(t) \int_t^\infty \frac{dt'}{a(t')} = a(t) \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}}. \tag{19}$$

The last integral has the explicit form as

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \frac{1}{\sqrt{|k|}} \text{sinn}^{-1}(\sqrt{|k|r_1}) = \begin{cases} \sin^{-1}(r_1), & k = 1, \\ r_1, & k = 0, \\ \sinh^{-1}(r_1), & k = -1. \end{cases} \tag{20}$$

The EoS parameter gives

$$\omega_{de}^{eff} = \frac{p_{eff}}{\rho_{de}} + \frac{\Gamma}{3H} = \frac{\chi}{\rho_{de}^{1+\alpha}} - 3vH\rho_{de}^{-1/2} + b^2 \frac{(1 + \Omega_k)}{\Omega_{de}}, \tag{21}$$

or we can write

$$\chi = \rho_{de}^{1+\alpha} \left[ \omega_{de}^{eff} + 3vH\rho_{de}^{-1/2} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right]. \tag{22}$$

Inserting (13) in (22), we obtain

$$\chi = (3c^2 M_p^2 L^{-2})^{1+\alpha} \left[ -\frac{1}{3} - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} + 3Hv(3c^2 M_p^2 L^{-2})^{-1/2} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right]. \tag{23}$$

Using (23) in (12), we get

$$D = a^{3(1-v\gamma)(1+\alpha)} (3c^2 M_p^2 L^{-2})^{1+\alpha} \left[ \frac{2}{3} - v\gamma - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} + 3Hv(3c^2 M_p^2 L^{-2})^{-1/2} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right]. \tag{24}$$

From (14) and (15), the kinetic term becomes

$$\begin{aligned} \dot{\phi}^2 &= \rho_\phi + p_\phi, = \frac{1}{\rho_{de}^\alpha} [\rho_{de}^{1+\alpha} + \chi - 3vH\rho_{de}^{\alpha+\frac{1}{2}}] \\ &= (3c^2 M_p^2 L^{-2}) \left[ \frac{2}{3} - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right]. \end{aligned} \tag{25}$$

Also, the potential term becomes

$$\begin{aligned} 2V(\phi) &= (\rho_{de} - p_{eff}) = \frac{1}{\rho_{de}^\alpha} (\rho_{de}^{1+\alpha} - \chi + 3\nu H \rho_{de}^{\alpha+1/2}) \\ &= (3c^2 M_p^2 L^{-2}) \left[ \frac{4}{3} + \frac{2\sqrt{\Omega_{de} - c^2 \Omega_k}}{3c} + b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right]. \end{aligned} \tag{26}$$

From (25), we have

$$\dot{\phi} = H M_p \left[ 3\Omega_{de} \left\{ \frac{2}{3} - \frac{2\sqrt{\Omega_{de} - c^2 \Omega_k}}{3c} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right\} \right]^{1/2}. \tag{27}$$

Using the relation with  $x = \ln a$  [67], we have

$$\dot{\phi} = \phi' H, \tag{28}$$

where prime denotes differentiation with respect to the e-folding time parameter  $x$ , we obtain

$$\phi' = M_p \left[ 3\Omega_{de} \left\{ \frac{2}{3} - \frac{2\sqrt{\Omega_{de} - c^2 \Omega_k}}{3c} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right\} \right]^{1/2}. \tag{29}$$

On integration, we get

$$\phi(a) - \phi(a_o) = \int_o^{\ln a} M_p \left[ 3\Omega_{de} \left\{ \frac{2}{3} - \frac{2\sqrt{\Omega_{de} - c^2 \Omega_k}}{3c} - b^2 \frac{(1 + \Omega_k)}{\Omega_{de}} \right\} \right]^{1/2}. \tag{30}$$

It is interesting to note that the above expressions for the kinetic and potential for the viscous dark energy are the ones as they were for the non-viscous case.

### 2.1 Interacting Holographic Viscous Dark Energy with Variable Newton’s Constant $G$

Now we perform the above analysis with considering variable  $G$  i.e.  $G = G(a)$  and  $G = G(t)$ . There is some evidence of a variable  $G$  through numerous astrophysical observations [68–70]. Models with variable  $G$  can fix some of the hardest problems in cosmology like the age problem, cosmic coincidence problem and determination of the precise value of the Hubble parameter [71–74].

Differentiating (16), we obtain

$$\dot{\rho}_{de} = -\rho_{de} \left( \frac{\dot{G}}{G} + 2\frac{\dot{L}}{L} \right). \tag{31}$$

Moreover, using the definitions  $\Omega_{de} = \rho_{de} / \rho_{cr}$  and  $\rho_{cr} = 3M_p^2 H^2$ , we can write

$$L = \frac{c}{H\sqrt{\Omega_{de}}}. \tag{32}$$

Using (32) in (31), we obtain

$$\dot{\rho}_{de} = -\rho_{de} H \left[ 2 - \frac{\sqrt{\Omega_{de}}}{c} \text{cosn} \left( \frac{\sqrt{|k|} R_h}{a} \right) + \frac{G'}{G} \right]. \tag{33}$$

Substituting backwards (33) in the energy conservation equation (6), we obtain

$$\omega_{de}^{eff} = -\left[\frac{1}{3} + \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c}\right] + \frac{G'}{3G}. \quad (34)$$

Making use of (34) in (22), we get

$$\begin{aligned} \chi = (3c^2M_p^2L^{-2})^{1+\alpha} & \left[ -\frac{1}{3} - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} \right. \\ & \left. + 3Hv(3c^2M_p^2L^{-2})^{-1/2} + \frac{G'}{3G} - b^2\frac{(1+\Omega_k)}{\Omega_{de}} \right]. \end{aligned} \quad (35)$$

Substituting (35) in (12), we get

$$\begin{aligned} D = a^{3(1-\nu\gamma)(1+\alpha)}(3c^2M_p^2L^{-2})^{1+\alpha} & \left[ \frac{2}{3} - \nu\gamma - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} \right. \\ & \left. + 3Hv(3c^2M_p^2L^{-2})^{-1/2} + \frac{G'}{3G} - b^2\frac{(1+\Omega_k)}{\Omega_{de}} \right]. \end{aligned} \quad (36)$$

Similarly, the kinetic and potential terms modify to

$$\dot{\phi}^2 = (3c^2M_p^2L^{-2}) \left[ \frac{2}{3} - \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} + \frac{G'}{3G} - b^2\frac{(1+\Omega_k)}{\Omega_{de}} \right], \quad (37)$$

$$2V(\phi) = (3c^2M_p^2L^{-2}) \left[ \frac{4}{3} + \frac{2\sqrt{\Omega_{de} - c^2\Omega_k}}{3c} + \frac{G'}{3G} + b^2\frac{(1+\Omega_k)}{\Omega_{de}} \right]. \quad (38)$$

Thus we have reconstructed the kinetic and potential terms for the viscous HDE which involve variation in  $G$ .

### 3 Conclusion

Holographic dark energy model presents a dynamical view of the dark energy which is consistent with the astrophysical observations. Different values of the holographic parameter  $c$  correspond to different values of the dark energy parameter  $\omega_{de}$  [75]. Thus it gives a nice connection between the two models. In this paper, we have constructed a correspondence between holographic dark energy and interacting dark energy. This formalism is made using the viscous generalized Chaplygin gas. This EoS belongs to a general class of inhomogeneous EoS as suggested in [10, 76]. Within the different candidates of dark energy, the Chaplygin gas has emerged as a possible unification of dark matter and dark energy, since its cosmological evolution is similar to an initial dust like matter and a cosmological constant for late times. We have found that the reconstruction of the kinetic and potential terms of the HDE are independent of the viscosity parameters. It implies that if the dark energy is of the holographic type then it will be non-viscous and non-dissipative. The viscosity effects at the cosmic scale, if any, will remain negligible in the evolution of holographic dark energy. Finally, we have constructed a similar correspondence by considering a variable  $G$ . It shows that the variable gravitational constant will modify the evolution of the scalar field while viscosity effects remain negligible.

**Acknowledgements** We would like to thank the referees for their useful criticism on this work.

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